Thermal Analysis of Long Tab Banded/Filled Cavity System

Lamtec long tab banded filled cavity roof system: standing seam roof panel, 3-5/16" roof clip, 8-1/2" z-purlin, foam thermal spacer block, R-11 fiberglass insulation without facing adjacent to roof panel, R-25 long tab faced bottom layer of fiberglass insulation.

Prepared by:
William Wangard, Ph.D.
and
Joseph G. Beard, P.E., Ph.D.

Engrana LLC
820 Davis Street, Ste. 122
Evanston, IL 60201
www.engrana.com
Tel: 847/424-2486

Prepared for:
Hal Robbins
Lamtec, Technical Director

Brad Heffelmire
Lamtec, Manager MBI Technical Support

Lamtec Corporation
5010 River Road
Mount Bethel, PA 18343
www.lamtec.com
Tel: 507/897-6081

June 18, 2014

SIGNATURES

1 Executive Summary

Lamtec's long tab banded/filled cavity roof system was analyzed using computational modeling to determine minimum amount of fiberglass batt insulation to achieve an overall assembly U-factor of 0.035 ($Btu/hr\ ft^2$ °F). The U-factor is the rate of heat energy transmitted through a unit area of construction, including boundary air films, due to a unit temperature difference. The modeling was accomplished with Comsol Multiphysics software which uses Finite Element Analysis (FEA) to solve the three dimensional heat transfer equation on geometries developed within Comsol. A Detailed description of the assembly and model inputs are presented in section 2 – Model Development. The approach used to solving the steady state heat transfer through the assembly, and calculation of the assembly U-factor, is detailed in section 3 – Methodology.

A U-factor of 0.035 ($Btu/hr\ ft^2$ °F) is achieved with an unfaced top insulation layer (adjacent to the roof panel) rated at R-11 and a long tabbed faced bottom layer filling the space between purlins rated at R-25. Using fiberglass reference properties from ASHRAE 90.1-2013 Table A9.4.5.1 gives a U-factor that ranges from 0.0342 to 0.0352 ($Btu/hr\ ft^2$ °F), dependent upon the size of air pockets within the assembly (i.e. gaps within the assembly not filled by insulation) and nominal loft and degree of compression of the insulation. Several small air pockets enhance the thermal performance resulting in a lower U-factor while greater compression of insulation at pinch points degrades performance, resulting in a higher U-factor.

2 Model Geometry

2.1 General Description and Model Extents

The Assembly has a metal exterior 24 gauge MR-24 roof panel fastened to a z-purlin that measures 8-1/2 inch high by 0.06 in thick (21.6 cm x 1.5 mm). Steel bands 1 x 0.022 in (2.54 cm x 0.6 mm) on 30 in (76.2 cm) centers are attached to the bottom of the z-purlins to support the bottom layer of R-25 insulation with long tabbed facing. The facing's tabs are routed up along the z-purlin and fixed to the top of the z-purlin. Unfaced R-11 insulation is installed perpendicular to the z-purlins on top of the bottom R-25 insulation. The SSR roof clips and foam thermal spacer blocks measuring 1 x 3 in. (2.5 x 7.6 cm) are installed on top of the unfaced R-11 insulation, directly above the roof purlins. The roof panels are installed in direct contact with the R-11 insulation and will compress the upper layer of fiberglass under the foam thermal spacer block.

The assembly is shown in Figures 1 and 2.

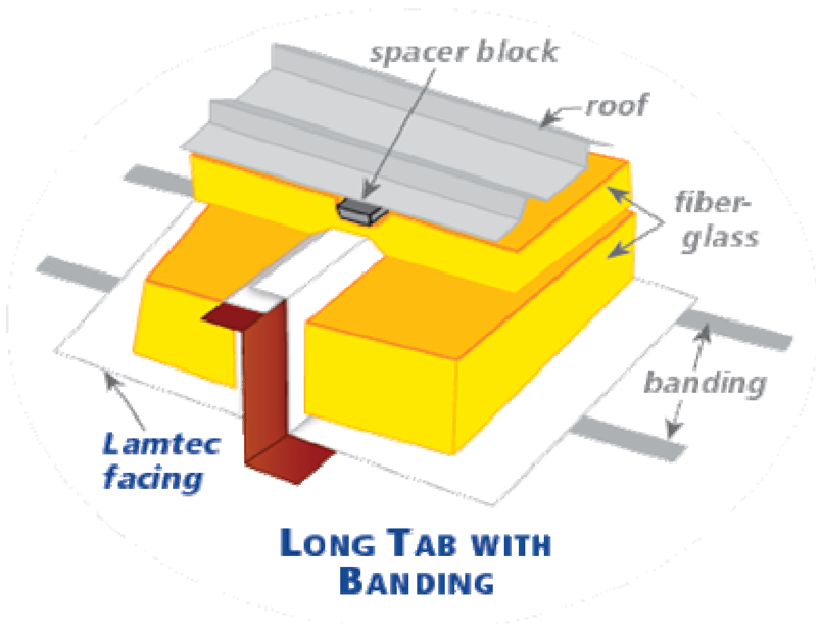

Figure 1 – Cutaway of roof assembly (courtesy of Lamtec).

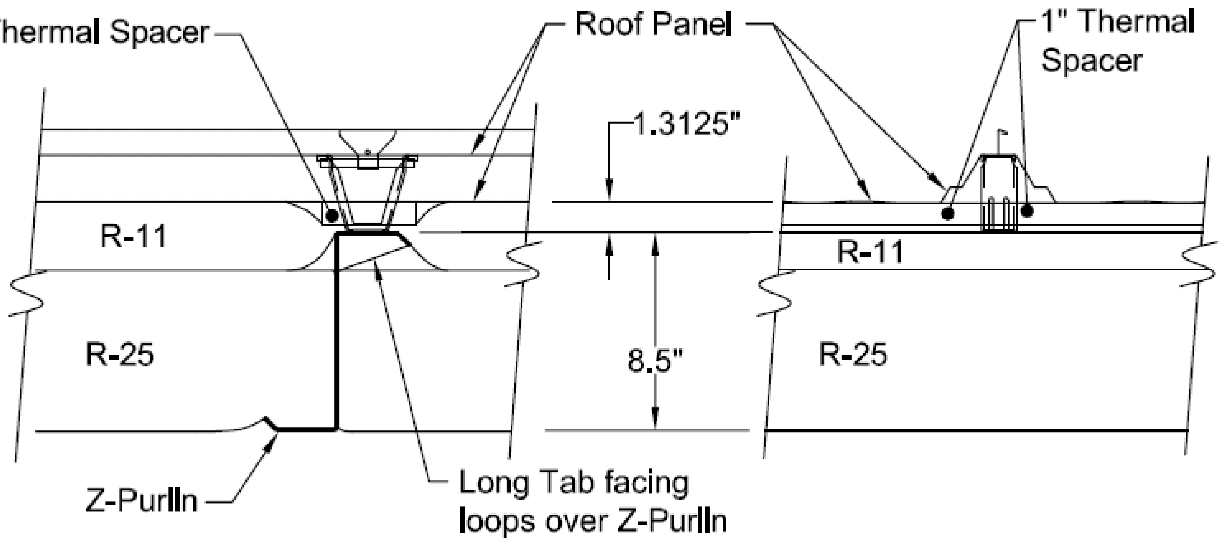

Figure 2 – Cross section of roof assembly.

2.2 Assembly Roof Panel

The modeled assembly uses the MR-24 roof panel. The panel is 24" (0.61 m) from center of ridge to center of ridge. The ridges are 2" high by 4.75" wide (5.1 by 12.1 cm). There is a 1.25" (3.175 cm) tab that runs down the center of each ridge that is the seam that joins the panels. This seam is not modeled and not shown in the figure below. Also neither shown nor modeled are 0.1" (2.5 mm) high flutes stamped into the panel perpendicular to the ridges. The flutes are 17" (43.2 cm) long and 6" (15.2 cm) on center. A 24 gauge panel is used in the model.

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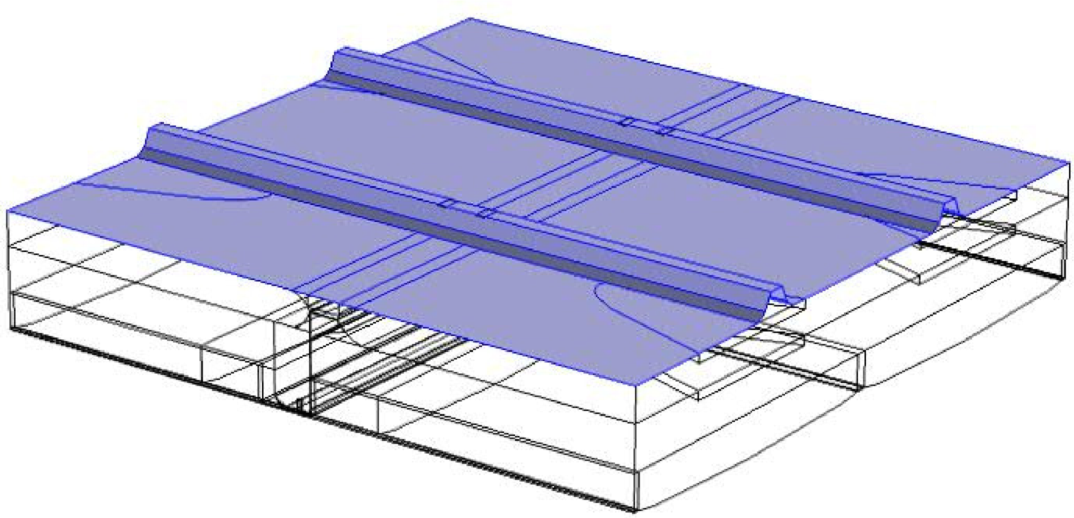

Figure 3 – MR-24 roof panel.

2.3 Purlins, and Panel Attachment

The z-purlin, MR-24 roof clip base, and steel banding are highlighted in the figure below. The model section of the assembly is five feet by five feet. The 0.06" thick (1.52 mm) z-purlins are on five foot centers and the modeled section has one z-purlin centered along one edge. The roof panel clip bases, 0.1 inch thick are atop the z-purlin and spaced on 24 inch centers to mate with the 24 inch spacing of the ridges in the roof panel system. The roof clip bases are 3.3125" (8.4 cm) high and attach to the roof panel at the panel's 2" (5.1 cm) high ridge, leaving 1.3125" (3.3 cm) between the roof panel and z-purlin. The steel bands attached to the bottom of the z-purlins are 1" wide by 0.022" thick (2.5 cm by 0.5 mm).

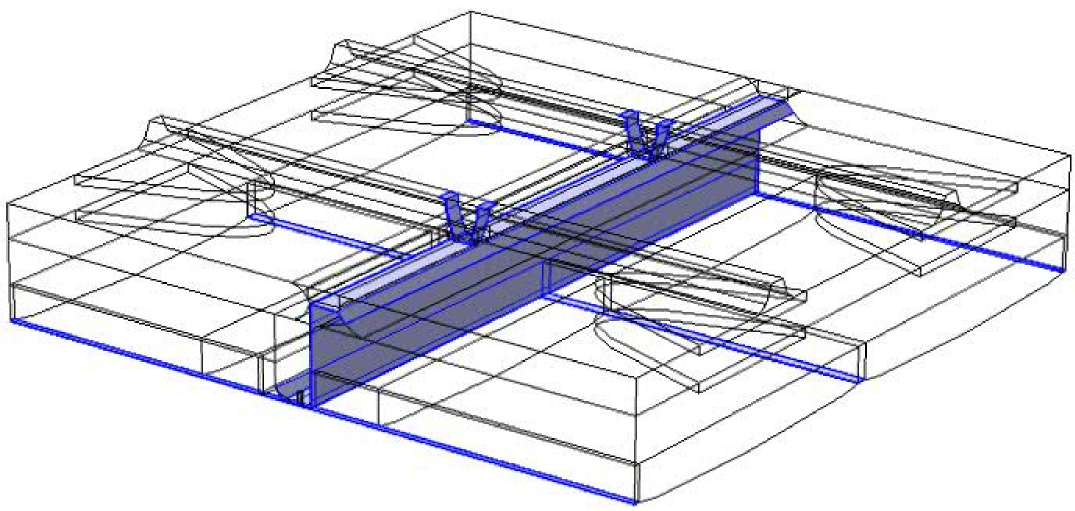

Figure 4 – Z-purlin, roof panel clip bases, and bottom steel banding.

2.4 Insulation

There is a top layer of R-11 and a bottom layer of R-25 fiber glass insulation, plus a foam thermal spacer block. The unfaced R-11 top layer is up against the roof panel laid perpendicular to, and over, the z-purlins. Where the R-11 crosses the purlins it is pinched between the foam thermal spacer block and the z-purlin as shown in figure 5 below. The bottom layer of R-25 is fitted between the z-purlins. The R-25 insulation has long tabbed facing, the edges of which run up against and over the z-purlins. The tabs from adjacent batts overlap on top of the purlin flange, both helping to secure the insulation and also maintaining the continuity of the vapor retarder (figures 6 and 7). The foam thermal spacer block is shown in figure 8.

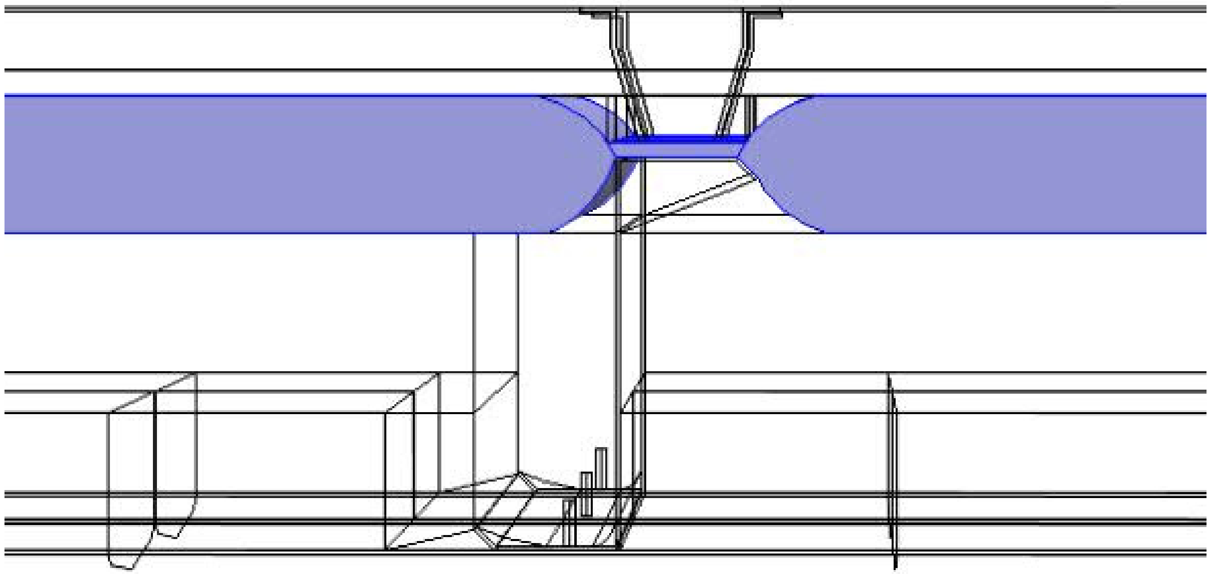

Figure 5 – Top insulation layer of R-11 showing compression between the z-purlin and foam thermal spacer block.

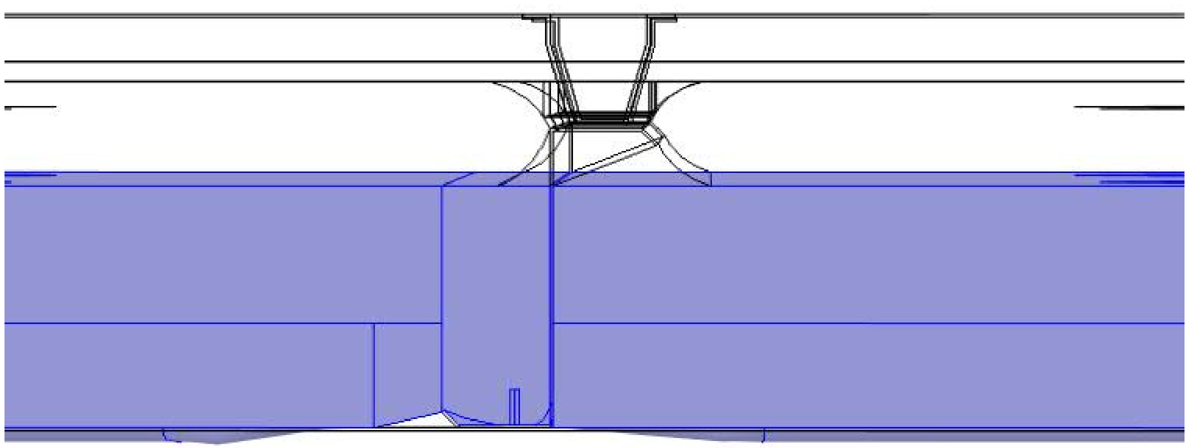

Figure 6 – Bottom insulation layer of R-25 showing compression by the z-purlin.

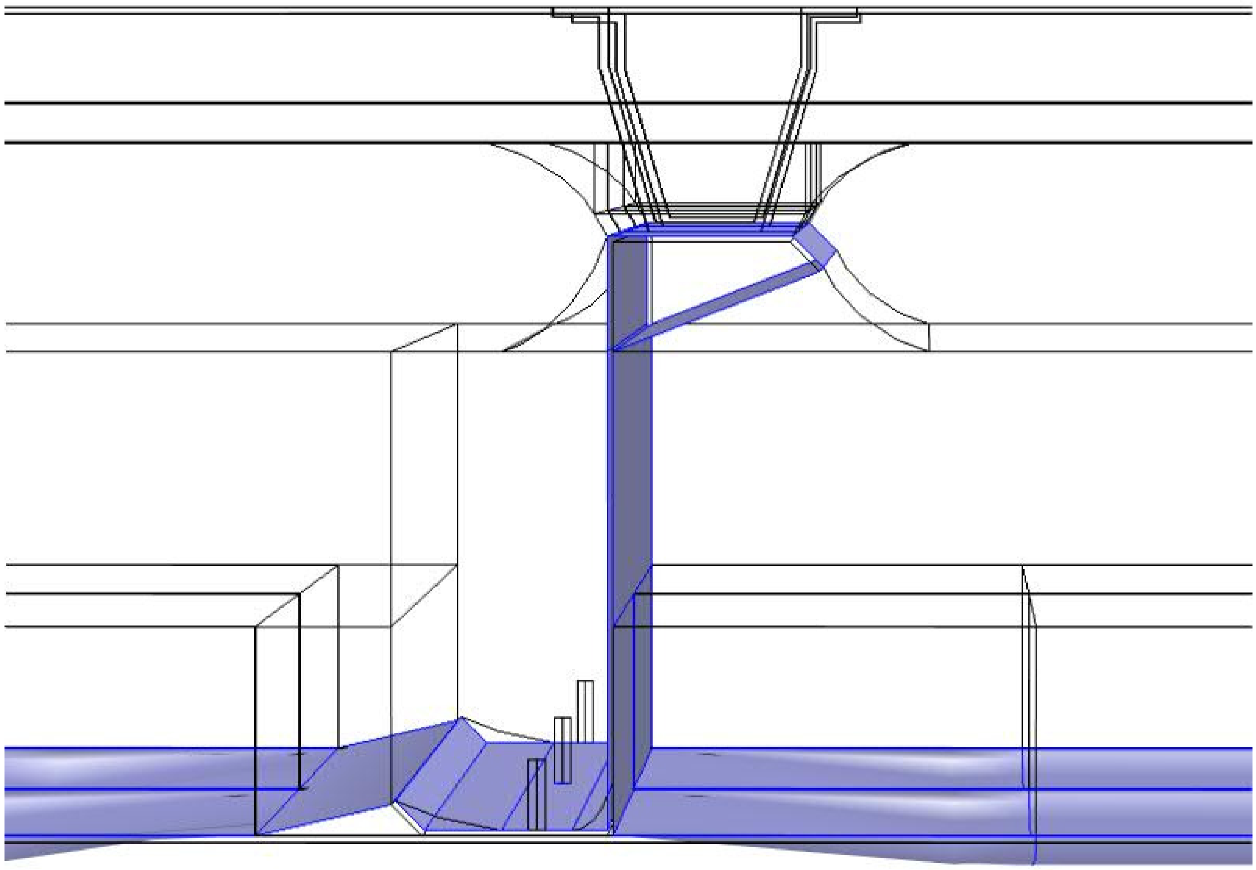

Figure 7 – Facing on R-25 insulation showing tabs over z-purlin.

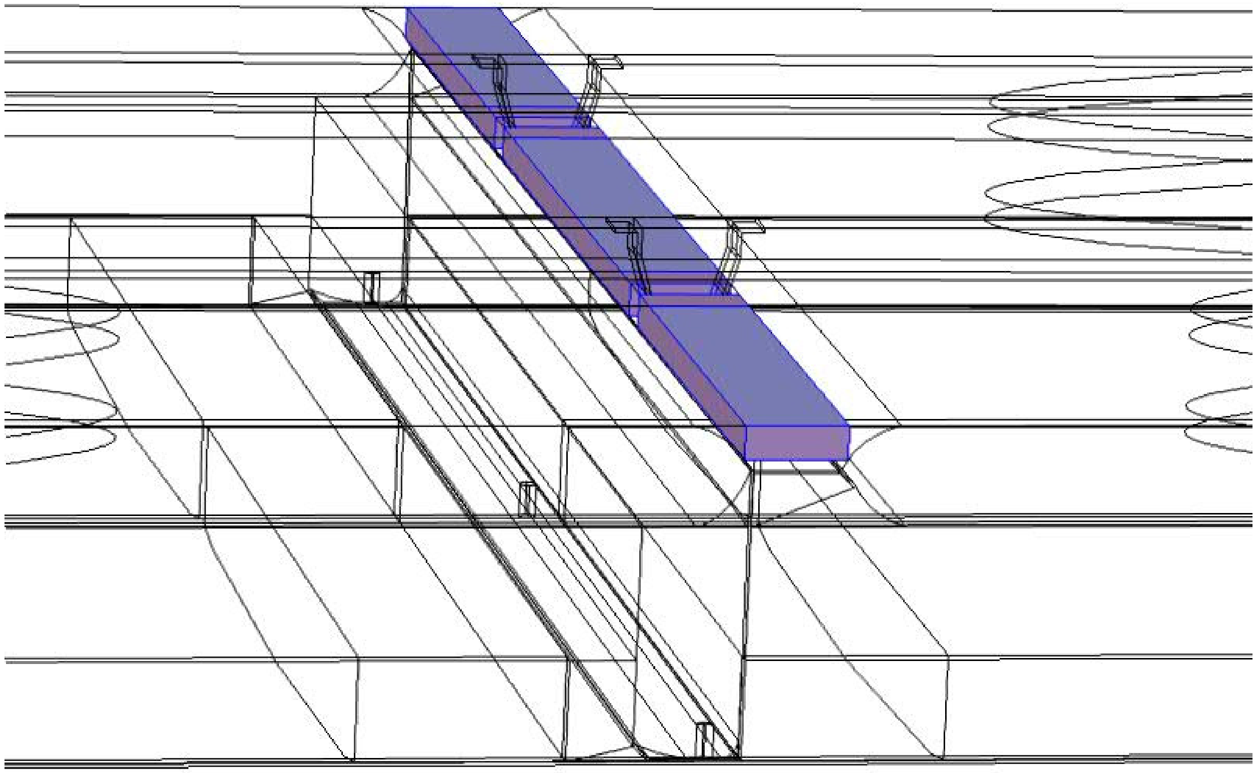

Figure 8 – Foam thermal spacer block.

The insulation properties are taken from ASHRAE 90.1-2013 §A9. For the R-11 and R-25 insulations at the ASHRAE loft, the density and conductivity are taken straight from Table A9.4 in the ASHRAE standard. For the loftier (thicker) insulation, the density is extrapolated so that the Wilkes equation in §3.3 gives the appropriate conductivity for the thickness listed. Those properties are given in the table below.

Table 1 – Fiber glass insulation properties.

Insulation	thickness t_0		density ρ_0		conductivity	
	cm	in	kg/m^3	lb/ft^3	W/m K	Btu in/hr ft^2 °F
ASHRAE R-11	8.1	3.20	10.8	0.674	0.0419	0.291
ASHRAE R-25	17.5	6.90	11.872	0.741	0.0397	0.275
loftier R-11	9.4	3.70	7.250	0.453	0.0479	0.332
loftier R-25	20.3	8.00	7.950	0.496	0.0460	0.319

2.5 Void Space

The void spaces in the model are defined and designated as air. The figure below shows the air spaces in the modeled assembly. There are spaces in the ridges of the roof panel, around the z-purlin, around the roof clip bases, and under the roof panel mid span (between purlins in one direction and bottom bands in the other) where the insulation is allowed to drape .

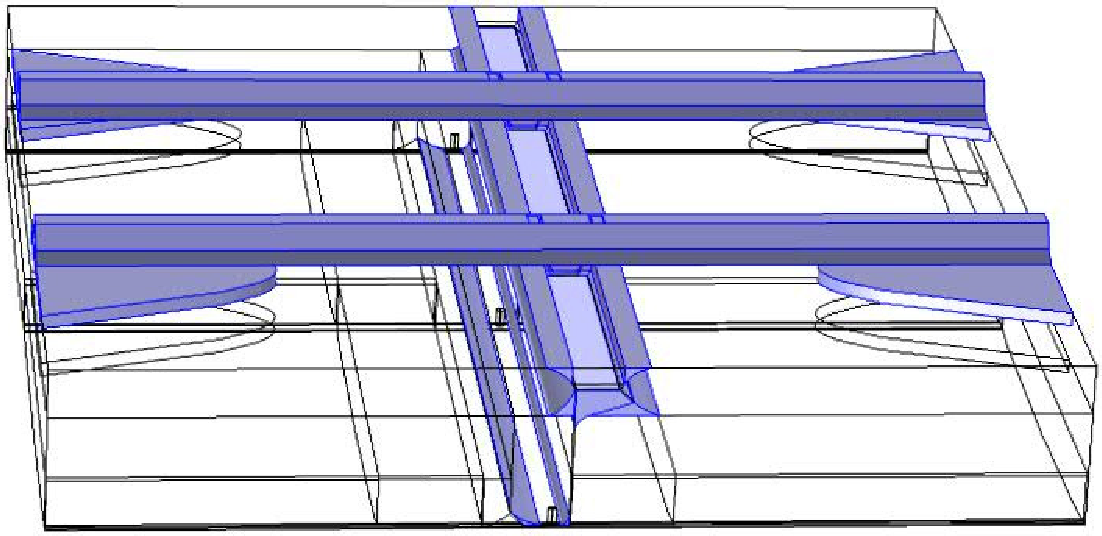

Figure 9a – Void spaces (air) in the modeled roof assembly with the lower U-factor.

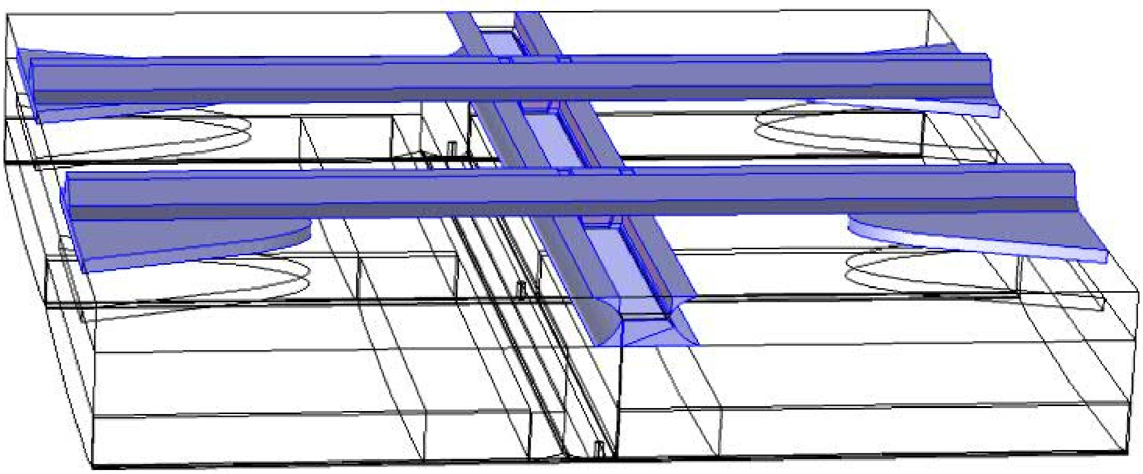

Figure 9b – Void spaces (air) in the modeled roof assembly with the higher U-factor.

3 – Methodology

Once the geometry has been defined, material properties are assigned, the equation to be solved is selected together with boundary conditions, and the assembly is meshed into discrete elements. Then the steady-state solution to heat transfer through the assembly is solved. Last, the U-factor is calculated by integrating the heat flux over either the top interior or exterior surface, and dividing the total heat flux at the surface by the projected area and the temperature difference applied in the boundary conditions.

3.1 The Conduction Equation

There are air spaces within the modeled assemblies where both convection and radiation could contribute to the heat transfer through the assembly. Also, radiative heat transfer plays a role within the fiberglass insulation. Only conduction, the primary mode of heat transfer within the air spaces, is modeled. Radiation and convection in the air spaces are not modeled for two reasons. First, the extent and size of the air spaces are poorly quantified so that the effects of convection and radiation within the spaces would be somewhat arbitrary. Second, and more important, is that the temperature difference that drives both convection and radiation are small across the air spaces so that inclusion of these modes of heat transfer would increase the complexity of the model without appreciably changing the results. It is worth noting that the conductivity of air is close to the conductivity of the fiberglass insulation so that errors in estimating air space dimensions will have a small impact on the overall heat transfer of the assembly.

With the radiative and convective modes of heat transfer removed from the model (except for the interior and exterior boundaries), the steady state heat equation reduces to the diffusion equation:

(3.1a)
$$\nabla \cdot \left(k\frac{\partial T}{\partial x}\hat{i} + k\frac{\partial T}{\partial y}\hat{j} + k\frac{\partial T}{\partial z}\hat{k} \right) = 0$$

with Neumann type boundary conditions

(3.1b)
$$k\nabla T = h_{exterior}\left(T_{exterior} - T_{surface}\right)$$

(3.1c)
$$k\nabla T = h_{\text{interior}}\left(T_{\text{interior}} - T_{surface}\right).$$

Where $T = f(x,y,z)$ is the temperature, $k = f(x,y,x)$ is the conductivity of the assembly, and the subscripts *exterior* and *interior* denote the exterior and interior surfaces (boundaries), respectively. Often, the conductivity is isotropic and the diffusion equation simplifies to $\nabla^2 T = 0$. However, in the models presented in this report, the conductivity, k, varies with position.

3.2 Material Properties

Before the conduction equation can be solved, the properties for each material in the assembly must be assigned. Properties for a given material can vary with composition and manufacture. Some properties, such as the steel, were taken from the FEA software material database. The conductivity of the fiberglass insulation is allowed to vary with its thickness. A detailed description of the fiberglass insulation conductivity is given in the next section. Specific heat (C_p) and density (ρ) are required by the FEA software for the solution to converge to steady-state, but any reasonable value suffices to achieve convergence without affecting the results, as steady-state heat transfer does not depend on either value.

Table 2 – Material Properties

Component	Material	Conductivity Btu in/hr ft^2 F	Density lb/ft^3
Purlin, clip base, banding	steel	309	490
Exterior panel	steel	309	490
Foam thermal spacer block	polystyrene	0.195	1.0
void space	air[1]	0.182	0.07
Insulation	Fiberglass[2]	0.290	0.6

[1] Air properties in table are at 300 K. The model uses variable properties from the Material Properties Database from JAHM Software

[2] Average conductivity at rated density

The convection coefficients for the inside and outside air films were calculated for hot box test conditions and are 7.42 and 7.33 W/m^{2}K (1.31 and 1.29 Btu/hr ft^2 °F) for the exterior and interior surfaces, respectively.

3.3 Insulation Conductivity

As long as fiberglass insulation is at or near its rated thickness, its nominal conductivity can be used as a homogenous material property. However, when the thickness varies significantly, as it does in metal building roof and wall assemblies, then a suitable expression for insulation conductivity should be employed. This section describes the expression selected for the fiberglass insulation conductivity used in the finite element modeling, how the expression is fit to the different insulation products, and how the density as a function of position in a modeled assembly is determined.

The apparent conductivity of fiberglass insulation varies with density, fiber diameter and orientation, temperature, and other parameters. A three coefficient expression of conductivity (k) as a function of density (ρ) was recommended Kenneth Wilkes [Wilkes 1979]. The Wilkes equation is

(3.2) $$k = A + B\rho + \frac{C}{\rho}$$

where A, B, and C are empirically determined constants. This work uses the conductivity curves presented in ASHRAE 90.1-2013 §A9 where A, B, and C are given as

A = 0.0149170 (English) and 0.00258168 (SI)

B = 0.0004377 (English) and 0.000047295 (SI)

C = 0.0056897 (English) and 0.157740033 (SI)

The nominal thicknesses and densities are given earlier in Table 1, §2.4.

To calculate the thickness anywhere within the fiberglass insulation, two equations are set up using the partial differential equation (PDE) solver within the finite element software. The thickness equations are:

(3.3a) $$\frac{\partial u}{\partial y} = 1$$

with a boundary condition that $u = 0$ along a selected insulation surface, and

(3.3b) $$\frac{\partial^2 v}{\partial y^2} = 0$$

with a boundary condition of $v = u$ along a selected insulation surface and zero flux $\left(\frac{\partial v}{\partial y} = 0\right)$ on all other surfaces.

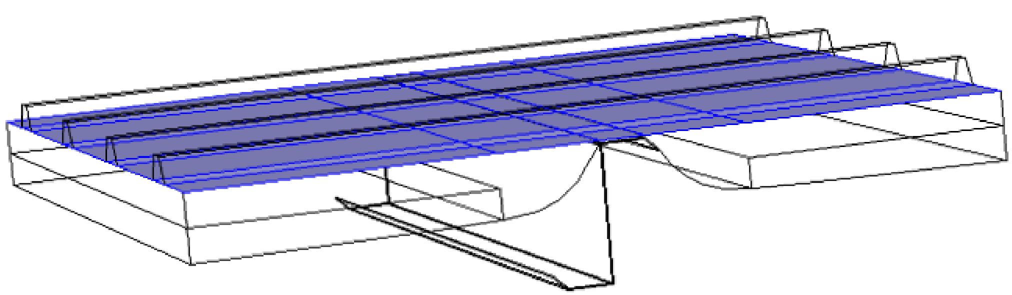

Figure 10a – Top boundary of insulation where $u = 0$. Assembly shown was not modeled in this work and is shown for illustrative purposes only.

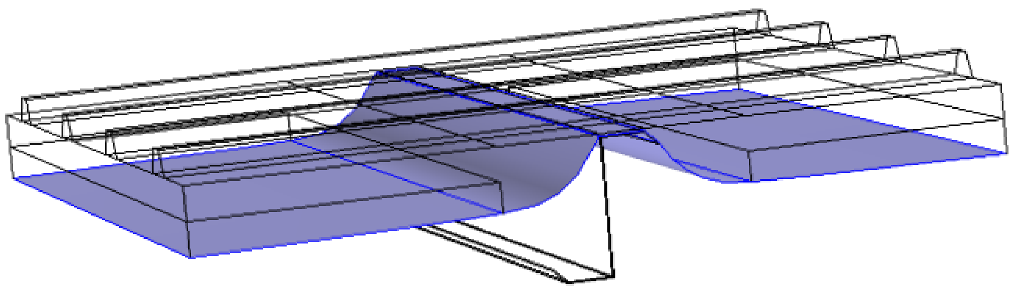

Figure 10b – Bottom boundary of insulation where $\left(\frac{\partial v}{\partial y}=0\right)$. Assembly shown was not modeled in this work and is shown for illustrative purposes only.

In equations 3.3a and 3.3b y is the independent variable because the models were set up with the y-coordinate perpendicular to the wall/roof panel.

The solution to 3.3a is $u = y + c_1$ and, since $u = 0$ on the top surface, the value of u is the distance along y measured from the top surface of the insulation. Specifically, the value of u at the bottom surface is the thickness of the insulation in the y-direction at any point (*x, z*). The second thickness equation, 3.3b, sets v equal to the thickness (*i.e.* u at the bottom surface).

While the general form of the solution to equation 3.3b is $v = c_2 y + c_3$, the zero flux condition at the top surface and the boundary condition at the bottom surface means $c_2 = 0$ and

$$v(x,z) = u(x,z)\Big|_{\text{y = bottom surface}}$$

Thus, at any point (*x, y, z*) the value of v equals the insulation thickness measured in the y-direction. The solution to the example case used here is given in the figure below.

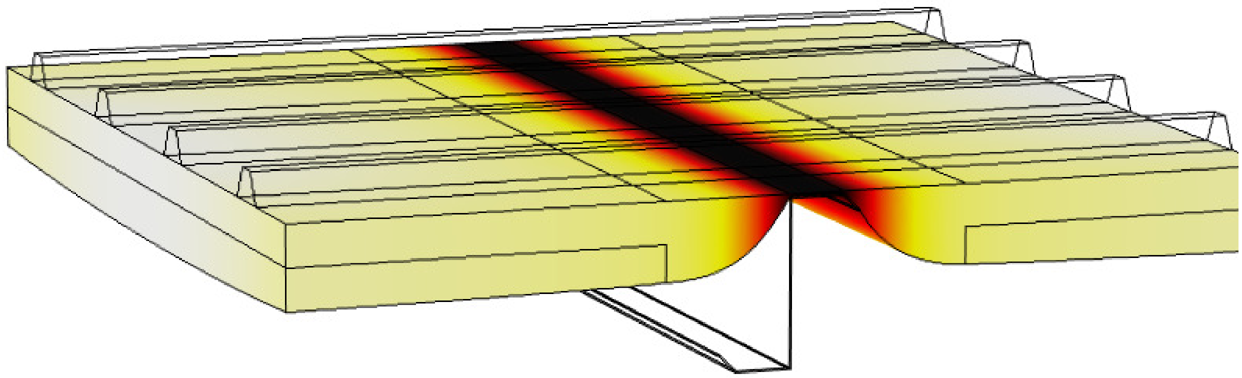

Figure 11 – Insulation thickness surface plot. Thickness ranges from to 0.125 in (0.3 cm) shown as darkest color to 4.5 in (11.4 cm) shown lightest. Assembly shown was not modeled in this work and is shown for illustrative purposes only.

3.4 Mesh

To solve the governing equation with the attendant boundary conditions (eq. 3.1), the geometries that comprise the modeled assembly must be meshed into discrete elements. The FEA software allows a great deal of flexibility in creating the mesh. For the assembly presented here, the auto-mesh was employed with tetrahedral elements. The auto-mesh has nine different mesh sizes to choose from. They are: extremely fine, extra fine, finer, fine, normal, coarse, coarser, extra coarse, and extremely coarse. The determination of which mesh size to use was made by increasing the resolution from coarse to fine until the further refinement had little effect on the results. The solution was computed on a normal the normal sized mesh shown below in Figure 12.

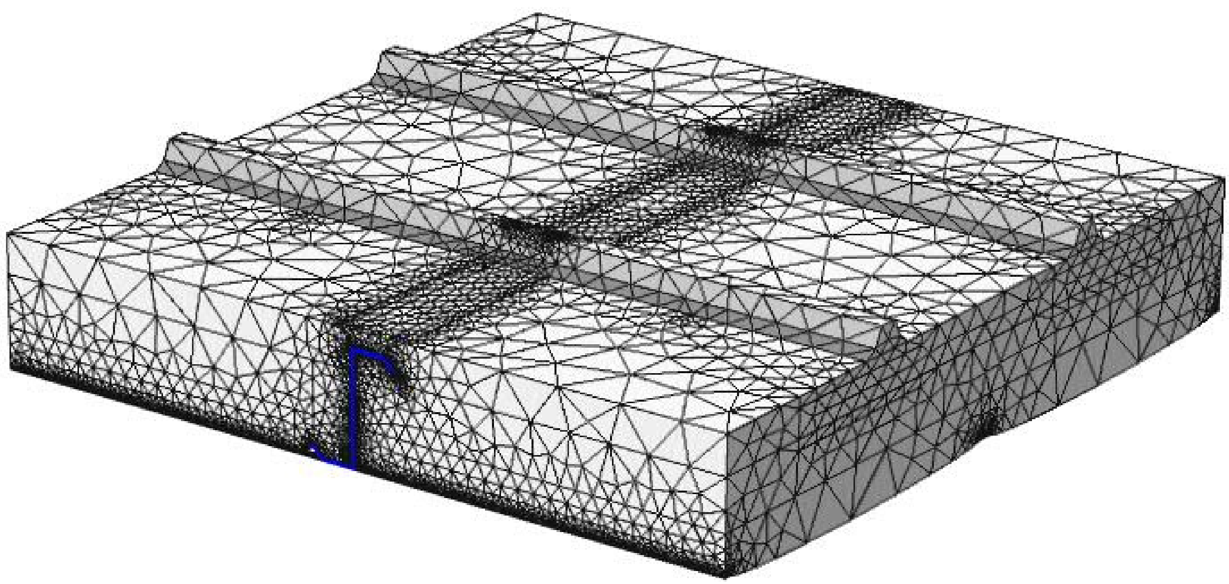

Figure 12 – Normal mesh on R-10 plus R-25 LTB assembly with 423,289 tetrahedral elements.

3.5 The U-factor

Since there is no heat storage in steady-state heat transfer, the total heat flux at either surface is the heat transferred through the assembly. Total heat flux, q'', is determined by evaluating:

(3.4a) $$q'' = q''_{exterior} = \int h_{exterior}\left(T_{exterior} - T_{surface}\right) dA_{surface}$$

or

(3.4b) $$q'' = q''_{\text{interior}} = \int h_{\text{interior}}\left(T_{\text{interior}} - T_{surface}\right) dA_{surface} \; .$$

Either 3.4a or 3.4b give the same result to five significant figures (at any mesh resolution). The boundary surface is greater than the projected surface because the interior and exterior boundaries are not flat. That is $A_{surface} \neq Area$ where *Area* is the projected area of the assembly for which the U-factor is to be calculated. The U-factor, then, is:

(3.3) $$U = \frac{q''}{Area \cdot \left(T_{\text{interior}} - T_{exterior}\right)}$$

The modeled roof assembly area = 2.3226 m^2 (25 ft^2. The temperatures were $T_{interior}$ = 310.9278 K (100 °F) and $T_{exterior}$ = 283.15 K (50 °F). Figure 11 gives the surface temperatures of the assembly. The calculated U-factor is 0.2 W/m^2 (0.035 Btu/hr ft^2 °F). There is, however a range of insulation lofts available for a given R-value rating. For a given R-value, a loftier fiberglass batt will suffer greater compression. Table 4 gives the U-factor for insulation lofts near the minimum and maximum for typical fiber glass batt insulation.

Table 4 – U-factor with two different insulation lofts.

R-11 plus R-25 insulation loft		U-factor	
inches	cm	W/m^2 K	Btu/hr ft^2 F
10.1	25.65	0.1940	0.0342
11.7	29.72	0.1998	0.0352

Signature: Joseph Beard
Joseph Beard (Jun 19, 2014)

Email: joseph@jgb-engineering.com

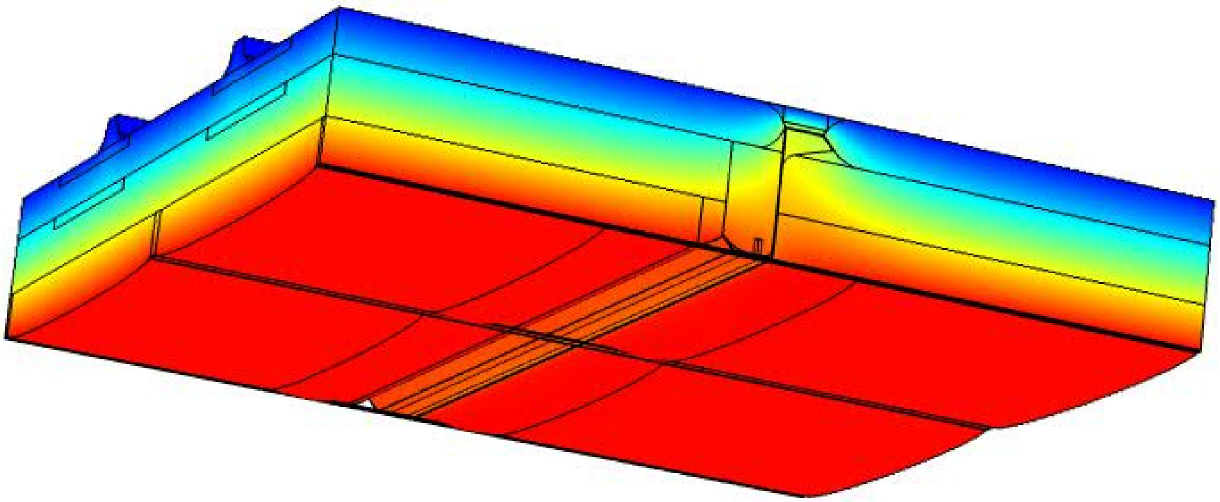

Figure 13 – Surface temperatures. Dark red = 310.93 K (100 °F) and dark blue = 283.15 K (50 °F).

Last, the computed U-factors are plotted below in figure 14 along with hot box test results for similarly constructed assemblies with various levels of insulation. The hot test results are a mixture of long tab banded systems and liner systems.

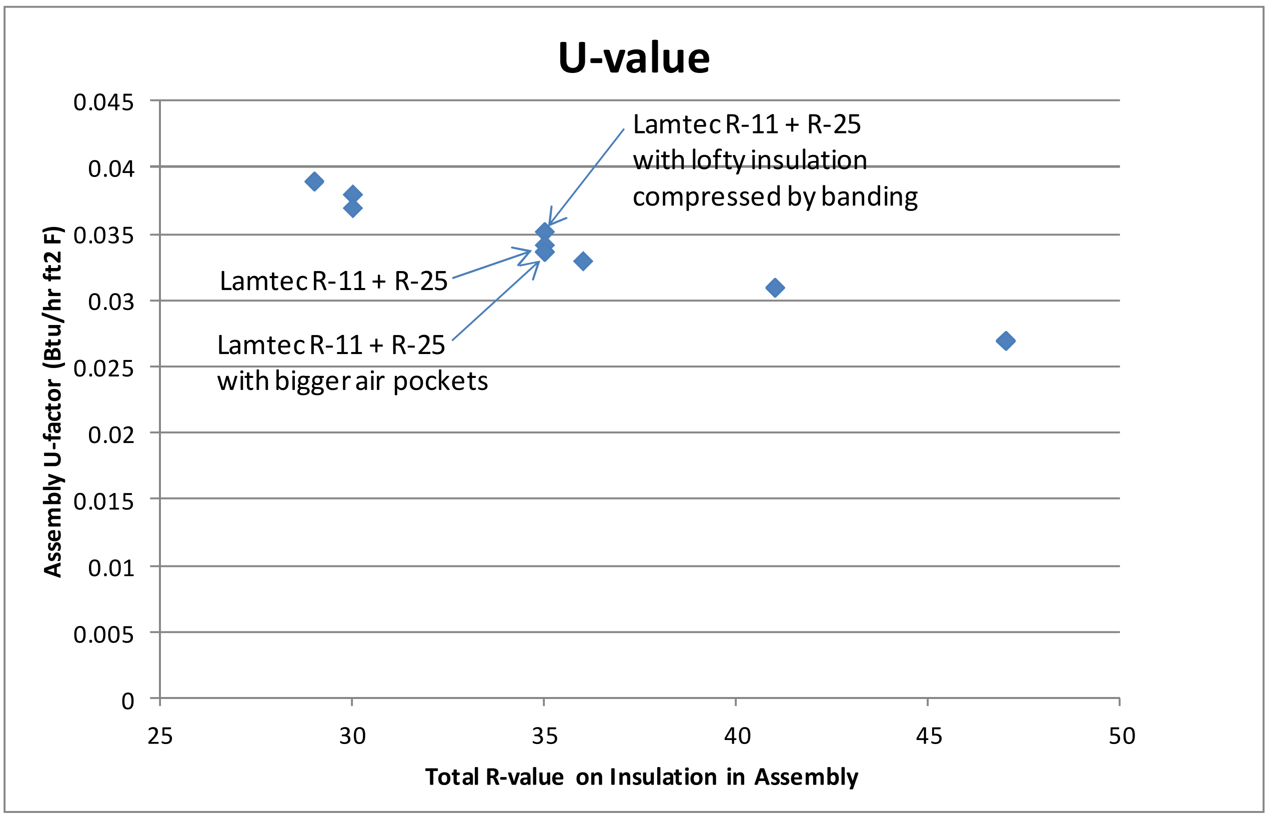

Figure 14 – Comparison of assembly U-factors of similar construction.